2714 [June, 1918]. Proposed by H. R. HOWARD, University of St. Francis Xavier's College, Nova Scotia.

A shuffled pack of $2(p+q)$ cards contains $2p$ honors. Show that the chance of securing exactly half the honors in taking half the pack is $[F(p, q)]^2 \div F(2p, 2q)$, where $F(p, q)$ denotes the number of different sets of p cards which can be selected from $(p+q)$ cards.

Show also that if one honor is removed from the pack, the chance is not thereby affected. Is this true for the chance of getting any other assigned number of honors?

SOLUTION BY THE PROPOSER.

We have to find first the number of ways in which we can get exactly p honors in $(p+q)$ cards.

We can obviously choose our "p" honors in $(2p)!/(p!\,p!)$ ways and the q other cards in $(2q)!/(q!\,q!)$ ways.

Hence, we can effect the required division in $[(2p)!\,(2q)!]/[(p!)^2\,(q!)^2]$ ways. Now the number of ways of taking $(p+q)$ cards from the full pack is $[2(p+q)]!/(p+q)!^2$. Thus the chance is

$$\frac{(2p)!\,(2q)!}{(p!)^2(q!)^2} \div \frac{[2(p+q)]!}{(p+q)!^2}, \tag{1}$$

i.e.,

$$\frac{(p+q)!^2}{(p!)^2\,(q!)^2} \div \frac{(2p+q)!}{(2p)!\,(2q)!},$$

i.e.,

$$[F(p, q)]^2 \div F(2p, 2q).$$

Now suppose one honor removed from the pack. Then the number of ways of taking $(p+q)$ cards from the remainder and obtaining exactly p honors is

$$\frac{(2p-1)!}{(p-1)!\,(p)!} \cdot \frac{(2q)!}{(q!)^2},$$

and the chance is

$$\frac{(2p-1)!}{(p-1)!\,(p)!}\,\frac{(2q)!}{(q!)^2} \div \frac{(2p+2q-1)!}{(p+q)!\,(p+q-1)!}. \tag{2}$$

Dividing (1) by (2) we obtain unity for the quotient and this proves their equality. Let x be the assigned number of honors. We shall show that the condition that the chances be equal can only be satisfied by $x = p$. With the full pack the chance will now be

$$\frac{(2p)!}{x!\,(2p-x)!} \cdot \frac{(2q)!}{(p+q-x)!\,(q-p+x)!} \div \frac{(2p+2q)!}{(p+q)!^2}. \tag{3}$$

With one honor removed the chance will be

$$\frac{(2p-1)!}{x!\,(2p-x-1)!}\,\frac{(2q)!}{(p+q-x)!\,(q-p+x)!} \div \frac{(2p+2q-1)!}{(p+q)!\,(p+q-1)!}. \tag{4}$$

Dividing (3) by (4) we see that these can only be equal if

$$\frac{p}{2p-x} = 1$$

or $x = p$, which proves our statement.

Also solved by H. L. OLSON, A. PELLETIER, and E. E. WITMER.

2715 [June, 1918]. Proposed by H. R. KINGSTON, University of Manitoba.

A', B', C' are points on the sides BC, CA, AB, respectively, of the triangle ABC, and AA', BB', CC' are concurrent in O. X, Y, Z are the three collinear points in which, by Desargues' theorem, the corresponding sides of the triangles ABC and $A'B'C'$ intersect. If A'', B'', C'' are the vertices of the triangle formed by the lines AX, BY, CZ, show that AA'', BB'', CC'' are concurrent.